Universidad Autónoma Metropolitana
Casa abierta al tiempo Azcapotzalco

*REPORTE DE INVESTIGACIÓN*

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**2. N. de proyecto registrado ante el Consejo Divisional:**
571, Tasa de Crecimiento en una Economía Liderada por el Sector Exportador.

**3. Línea de Generación y/o Aplicación de Conocimiento:** Crecimiento Económico

**4. Área, Grupo de investigación:** Modelación económica, teórica y aplicada (en aprobación)

**A) Título**:

**TRANSITIONAL DYNAMICS, EXTERNALITIES, OPTIMAL SUBSIDY AND GROWTH**

**B) Resumen:**

Se desarrolla un modelo de crecimiento endógeno con dos sectores, el sector manufacturero (learning) y el no manufacturero (no-learning). Se supone que el sector manufacturero es el único sector que genera conocimiento tecnológico nacional a través de aprender hacienda (learning by doing). El conocimiento que se produce en el sector manufacturero está disponible para el sector no manufacturero. Se obtienen las funciones de política para la economía de mercado y la economía del planificador. Así, con la solución óptima, se obtiene la trayectoria temporal del subsidio a la inversión óptimo. El subsidio a la inversión óptimo está aumentando, mientras que la economía de mercado se mueve al estado estacionario óptimo.

We develop an endogenous growth model with two sectors, manufacturing (learning) and non-manufacturing (non-learning). We assume that the manufacturing sector is the only sector that generates domestic technological knowledge through learning by doing. The knowledge produced in the manufacturing sector is available to the non-manufacturing sector. We obtain policy functions for the market economy and the command economy. Thus, with the optimal solution, we obtain the time path of the optimal investment subsidy. The optimal investment subsidy is increasing while the market economy moves to the optimal steady state.

**C) Introducción:**

In this paper, we study the relationship between subsidies and economic growth with a multi-sector dynamic general equilibrium approach.

Consequently, we develop an endogenous growth model with two sectors, manufacturing (learning) and non-manufacturing (non-learning), with two types of capital. We assume that the manufacturing (learning) sector is the only sector that generates domestic technological knowledge through learning by doing. The knowledge produced in the manufacturing sector is available to the non-manufacturing (non-learning) sector. Thus, the model has two learning externalities. Therefore, the manufacturing sector drives the market economy to a sustained positive growth rate. We assume that the two goods are consumed and accumulated. The government taxes households with a lump-sum tax to finance an investment subsidy in the manufacturing sector. Households own both types of capital. The main objective of this paper is to obtain the optimal subsidy in the steady state and in the transition. Our model is related to dependent economy models with two types of capital and externalities. Thus, Brock and Turnovsky (1994), and Turnovsky (1996) develop models with two types of capital. In particular, Korinek and Serven (2010) develop an endogenous growth model where the tradable sector generates higher learning externalities than the non-tradable sector.

First, we present a market economy with zero subsidies. With the aim of identifying the optimal subsidy, we find the planner´s solution where both externalities are internalized. Thus, with the optimal solution, we obtain the optimal rate of investment subsidy to the manufacturing sector in the market economy. First, we study how the economy responds, in the steady state, when the government establishes the optimal rate of investment subsidy in the manufacturing sector. Thus, when the subsidy is increased, the manufacturing sector is encouraged, and the proportion of labor in the manufacturing sector increases initially. Likewise, investment in the manufacturing sector expands, and investment in the non-manufacturing sector falls. Consequently, the ratio of non-manufacturing to manufacturing capital decreases slowly. Given that the price of the non-manufacturing good is determined by supply and demand, the relative price of the non-manufacturing good decreases initially. This produces an additional initial increase in the proportion of labor in the manufacturing sector. However, the level of the relative price of the non-manufacturing good is higher in the optimal steady state. Moreover, as total wealth increases, the ratio of consumption to non-manufacturing capital increases, as well. Therefore, since the manufacturing sector is the leader in technological terms, the market economy has a higher long run growth rate.

Next, in order to study the transitional dynamics of the economy, we use the time-elimination method (see Mulligan and Sala-i-Martin, 1991 and 1993). We obtain policy functions for the market economy, that is, functional relationships between control and state variables. With the aim of identify the time path of the optimal subsidy, we obtain policy functions for the command economy. Thus, with the optimal solution, we obtain the time path of the optimal investment subsidy to the manufacturing sector in the market economy. The time path of the optimal investment subsidy is increasing while the economy moves to the optimal steady state.

Thus, we have generalized in an economy with two learning externalities, two capital goods, and endogenous growth, the basic conclusion of the learning-by-doing literature that the first best response of the government is to establish an investment subsidy in the learning sector. Thus, the optimal policy is to encourage the sources of the learning process, the manufacturing sector (see Clemhout and Wan, 1970; Bardhan, 1971; Succar, 1987; Boldrin and Scheinkman, 1988; Young, 1991; Rauch, 1992; and Aizenman and Lee, 2010). Therefore, the results of the impact of the subsidy on the relative price, the allocation of labor between sectors and growth that we have obtained are not present in the literature and contribute to a better understanding of the relationship between subsidies and economic growth. However, whether subsidies are permitted or not, or whether governments have the ability to deal appropriately with externalities or not, it still remains to be discussed as to what extent these subsidy processes can be carried out in a democratic country; that is, how a government can justify subsidizing one particular sector. These questions belong to the arena of political economy.

In section 2, we develop a model of a competitive market economy. We construct a system of differential equations describing the economy. We study the transitional dynamics of the market economy and we obtain the policy functions. In section 3, we discuss the planner´s solution and we conclude that the optimal growth rate is higher than that achieved in the market economy. In section 4, we deduce the time path of the optimal investment subsidy to the manufacturing sector. In section 5, we present our conclusions.

**D) Desarrollo:**

## 2. THE COMPETITIVE MARKET ECONOMY

In this section, we develop a dynamic general equilibrium model of a competitive market economy. There are two production sectors, the manufacturing (learning) and non-manufacturing (non-learning) sectors. There are a large number of competitive manufacturing and non-manufacturing firms with the same production function. The manufacturing good and the non-manufacturing good are produced, accumulated, and consumed. The output in each sector is produced through physical capital, labor, and technological knowledge. The total labor supply is constant. Labor is freely mobile between the two sectors. The representative household maximizes the present value of a utility function. The consumption basket is formed by the manufacturing and non-manufacturing goods. The government collects taxes and gives subsidies.

### 2.1 THE MANUFACTURING SECTOR

We assume that the production function of the manufacturing (learning) firm $i$ ($i = 1, \ldots, N$, where $N$ is large) is Cobb-Douglas:

$$Y_{M_i} = A_{M_i} K_{M_i}^{\alpha} L_{M_i}^{1-\alpha} E_1$$

where $Y_{M_i}$ is the output of the manufacturing firm $i$, $A_{M_i}$ is a positive parameter of efficiency, $K_{M_i}$ is the stock of physical capital accumulated of the manufacturing good in the manufacturing firm $i$, $L_{M_i}$ is the labor employed in the manufacturing firm $i$, $\alpha$ and $1-\alpha$ are the shares of $K_{M_i}$ and $L_{M_i}$, respectively, with $0 < \alpha < 1$, and $E_1$ is a learning externality.

Let $K_M$ be the aggregate stock of physical capital accumulated of the manufacturing good. Domestic technological knowledge is created through learning by doing in the manufacturing sector, so knowledge is a by-product of investment (Arrow, 1962). Since knowledge is a public good, there are spillover effects of knowledge across manufacturing firms. Therefore, $E_1$ is the external effect of $K_M$ in the production function of the manufacturing firm $i$. In order to generate endogenous growth, we assume $E_1 = K_M^{1-\alpha}$ (Romer, 1986, Romer, 1989).

Given that all the manufacturing firms make the same choice, we obtain the aggregate production function of the manufacturing sector:

$$Y_M = A_M K_M^{\alpha} L_M^{1-\alpha} [K_M^{1-\alpha}] \quad (1)$$

where $Y_M$ is the aggregate output in the manufacturing sector, $A_M$ is the aggregate positive parameter of efficiency, and $L_M$ is the aggregate labor employed in the sector. We assume that $K_M$ is used only in the manufacturing sector.

Considering that the rate of depreciation of $K_M$ is zero and that the price of the manufacturing good is the *numéraire*, the rental price of $K_M$ is $R_M = r$, where $r$ is the interest rate. As we will see, the optimal government policy is to establish an investment subsidy in the manufacturing sector. Thus, we introduce a rate of investment subsidy, $\mu$, where $0 < \mu < 1$. Taking the externality as given, the manufacturing firms maximize profits $\pi_M = A_M K_M^{\alpha} L_M^{1-\alpha} [K_M^{1-\alpha}] - w_M L_M - R_M (1-\mu) K_M$, where $w_M$ is the wage rate in the sector. The first order conditions are:

$$w_M = A_M K_M^{\alpha} (1-\alpha) L_M^{-\alpha} [K_M^{1-\alpha}] = A_M K_M (1-\alpha) L_M^{-\alpha} \quad (2)$$

$$R_M (1-\mu) = r(1-\mu) = A_M \alpha K_M^{\alpha-1} L_M^{1-\alpha} [K_M^{1-\alpha}] = A_M \alpha L_M^{1-\alpha} \quad (3)$$

Equation (2) states that the wage rate is equal to the value of the marginal product of $L_M$. Equation (3) states that the interest rate, net of subsidy, is equal to the marginal product of $K_M$.

## 2.2 THE NON-MANUFACTURING SECTOR

We assume that the production function of the non-manufacturing (non-learning) firm $i$ is Cobb-Douglas:

$$Y_{N_i} = A_{N_i} K_{N_i}^{\beta} L_{N_i}^{1-\beta} E_2$$

where $Y_{N_i}$ is the output of the non-manufacturing firm $i$, $A_{N_i}$ is a positive parameter of efficiency, $K_{N_i}$ is the stock of physical capital accumulated of the non-manufacturing good in the non-manufacturing firm $i$, $L_{N_i}$ is labor employed in the non-manufacturing firm $i$, $\beta$ and $1-\beta$ are the shares of $K_{N_i}$ and $L_{N_i}$, respectively, with $0<\beta<1$, and $E_2$ is a learning externality. Since there are spillover effects of knowledge between the sectors, $E_2$ is technological knowledge generated in the manufacturing sector, but used in the non-manufacturing sector. We consider that these inter-industry benefits of knowledge are purely external to the non-manufacturing firm $i$. Thus, $E_2$ is the external effect of $K_M$ in the production function of the non-manufacturing firm $i$. We assume $E_2 = K_M^{1-\beta}$.

Given that all the non-manufacturing firms make the same choice, we obtain the aggregate production function of the non-manufacturing sector:

$$Y_N = A_N K_N^{\beta} L_N^{1-\beta} \left[K_M^{1-\beta}\right] \quad (4)$$

where $Y_N$ is the aggregate output in the non-manufacturing sector, $A_N$ is the aggregate positive parameter of efficiency, $K_N$ is the aggregate stock of physical capital accumulated of the non-manufacturing good, $L_N$ is the total labor employed in the non-manufacturing sector. We assume that $K_N$ is used only in the non-manufacturing sector.

We define $p_N$ as the relative price of the non-manufacturing to the manufacturing good. Considering that the rate of depreciation of $K_N$ is zero, the rental price of $K_N$ is $R_N = p_N(r - \dot{p}_N / p_N)$, where $\dot{p}_N / p_N$ is the growth rate of $p_N$ (capital gains of $K_N$). Taking the externality as given, the non-manufacturing firms maximize profits $\pi_N = p_N A_N A_N K_N^{\beta} L_N^{1-\beta} [K_M^{1-\alpha}] - w_N L_N - R_N K_N$, where $w_N$ is the wage rate in the sector. The first order conditions are:

$$w_N = p_N A_N K_N^{\beta}(1-\beta)L_N^{-\beta}\left[K_M^{1-\beta}\right] = p_N A_N K_N^{\beta} K_M^{1-\beta}(1-\beta)L_N^{-\beta} \quad (5)$$

$$R_N = p_N(r - \dot{p}_N/p_N) = p_N A_N \beta K_N^{\beta-1} L_N^{1-\beta}\left[K_M^{1-\beta}\right] = p_N A_N \beta K_N^{\beta-1} K_M^{1-\beta} L_N^{1-\beta} \quad (6)$$

Equation (5) states that the wage rate is equal to the value of the marginal product of $L_N$. Equation (6) states that the rental price of $K_N$ is equal to the marginal product of $K_N$ or the interest rate is equal to the marginal product of $K_N$ plus capital gains.

**2.3 THE GOVERNMENT**

The investment subsidy is financed through lump-sum taxes, $T$, to the households. The government has a balanced government budget constraint:

$$T = \mu R_M K_M \quad (7)$$

where $\mu R_M K_M$ is the amount of investment subsidy in the manufacturing sector.

**2.4 THE REPRESENTATIVE HOUSEHOLD**

The household disposable income is the sum of labor income and interest on assets less lump-sum taxes. This disposable income is allocated to consumption or saving. Thus, the budget constraint of the representative household is:

$$w_M L_M + w_N L_N + R_M K_M + R_N K_N - T = C_M + p_N C_N + I_M + p_N I_N \quad (8)$$

where $w_M L_M + w_N L_N$ is wage income, $R_M K_M + R_N K_N$ is capital income, $C_M$ is consumption of the manufacturing good, $C_N$ is consumption of the non-manufacturing good, $I_M = \dot{K}_M$ is the net investment in $K_M$, and $I_N = \dot{K}_N$ is the net investment in $K_N$. Next, we can define $C$ (aggregate consumption) as a homothetic index of $C_M$ and $C_N$: $C = DC_M^{\gamma}C_N^{1-\gamma}$, where $D = \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)}$ is a parameter, and $\gamma$ and $1-\gamma$ are the shares of $C_M$ and $C_N$ in the total expenditure on consumption, respectively, with $0<\gamma<1$. The consumer price index, $p_C$, is defined as $p_C = p_N^{1-\gamma}$. Thus, the total expenditure on consumption is:

$$p_C C = C_M + p_N C_N \tag{9}$$

Households can borrow and lend in the debt market (zero net loans in the aggregate). Also, we define $A = K_M + p_N K_N$, where $A$ are assets, and $\dot{A} = \dot{K}_M + p_N\dot{K}_N + \dot{p}_N K_N$. Using the previous concepts, the budget constraint, equation (8), becomes:

$$w_M L_M + w_N L_N + rA - T = p_C C + \dot{A} \tag{10}$$

The decision problem of the representative household is to choose a path of aggregate consumption that maximizes the present value of a utility function with a constant elasticity of intertemporal substitution, $\sigma$, and a constant subjective discount factor, $\rho$, where $\rho > 0$:

$$\max U(0) = \int_0^{\infty} \frac{C^{1-1/\sigma}}{1-1/\sigma} e^{-\rho t} dt$$

where $C = DC_M^{\gamma}C_N^{1-\gamma}$, subject to the budget constraint, equation (10), and to the solvency condition $\lim_{t\to\infty} Ae^{-\int_0^t r_v dv} \geq 0$.

The first order conditions are:

$$\frac{\dot{\lambda}_C}{\lambda_C} = \rho - r \tag{1}$$

$$\lambda_C = \frac{C^{-\frac{1}{\sigma}}}{p_C} \tag{2}$$

and $\lim_{t\to\infty} \lambda_C e^{-\rho t} A = 0$, where $\lambda_C$ is the shadow price, as of time $t$, of $A$ at time $t$. Next, considering that $p_N$ varies with time, we take logarithms and time derivatives of the consumer price index and obtain:

$$\frac{\dot{p}_C}{p_C} = (1-\gamma)\frac{\dot{p}_N}{p_N} \tag{3}$$

Also, we take logarithms and time derivatives of equation (12) and obtain:

$$\frac{\dot{\lambda}_C}{\lambda_C} = -\left(\frac{1}{\sigma}\right)\frac{\dot{C}}{C} - \frac{\dot{p}_C}{p_C} \tag{4}$$

Substituting equations (11) and (13) in (14), we obtain the dynamic allocation condition for aggregate consumption:

$$\frac{\dot{C}}{C} = \sigma\left[r - (1-\gamma)\frac{\dot{p}_N}{p_N} - \rho\right] \tag{5}$$

The optimal consumption basket of $C_M$ and $C_N$ results from static maximization of the utility function $DC_M^{\gamma}C_N^{1-\gamma}$ subject to the total expenditure on consumption, equation (9). The static first order conditions are:

$$C_M = \gamma p_C C \tag{16}$$

$$C_N = (1-\gamma)\frac{p_C C}{p_N} \quad (6)$$

## 2.5 EQUILIBRIUM IN GOODS AND LABOUR MARKETS

We can now proceed to obtain the resource constraint of the economy. Substituting equations (2), (3), (5), (6) and (7) in the budget constraint of the representative household, equation (8), we obtain

$$Y_M + p_N Y_N = C_M + p_N C_N + I_M + p_N I_N \quad (18)$$

Equation (18) is the aggregate equilibrium condition for the goods market, where the value of the total output, $Y$, is $Y = Y_M + p_N Y_N$. Next, we define the equilibrium condition for the non-manufacturing good market. The relative price of the non-manufacturing good is flexible, ensuring that the supply of the non-manufacturing good is always equal to its demand:

$$Y_N = C_N + I_N \quad (19)$$

With the equilibrium condition for the non-manufacturing good market, equation (18) becomes:

$$Y_M = C_M + I_M \quad (20)$$

The equilibrium condition in the labor market is:

$$L_M + L_N = L \quad (21)$$

where $L$ is total labor supply and we assume that it is constant.

## 2.6 THE MODEL IN STATIONARY VARIABLES

Given that $C$, $K_M$, $K_N$, $Y_M$, $Y_N$ and $Y$ are growing at all times, to solve the model we define the variables in terms of stationary variables. The characteristic of these variables is that they remain constant in the steady state (see Barro and Sala-i-Martin, 2004). Thus, we define the variables $z = K_N/K_M$ and $v = C/K_N$ as stationary variables. As $L$ is constant, it is normalized to one. Thus, the equilibrium condition in the labor market is $n+(1-n)=1$, where $n$ is the fraction of labor employed in the manufacturing sector, and $(1-n)$ is the fraction of labor employed in the non-manufacturing sector. Given that $n$ is constant in the steady state, we can use it as another stationary variable. Therefore, we can rewrite the aggregate production functions as:

$$Y_M = A_M K_M n^{1-\alpha} \tag{22}$$

$$Y_N = A_N K_M z^{\beta} (1-n)^{1-\beta} \tag{23}$$

We can rewrite the first order conditions (2), (3), (5) and (6) as:

$$w_M = A_M K_M (1-\alpha) n^{-\alpha} \tag{24}$$

$$r(1-\mu) = A_M \alpha n^{1-\alpha} \tag{25}$$

$$w_N = p_N A_N K_M z^{\beta} (1-\beta)(1-n)^{-\beta} \tag{26}$$

$$r - \frac{\dot{p}_N}{p_N} = \frac{A_N \beta (1-n)^{1-\beta}}{z^{1-\beta}} \tag{7}$$

Equating the value of the marginal product of labor in both sectors, equations (24) and (26), we find the static efficient allocation condition for labor between the sectors:

$$A_M (1-\alpha) n^{-\alpha} = p_N A_N z^{\beta} (1-\beta)(1-n)^{-\beta} \tag{28}$$

With equations (25) and (27), we obtain the dynamic arbitrage condition for the two capital goods:

$$\frac{A_M \alpha n^{1-\alpha}}{(1-\mu)} = \frac{A_N \beta (1-n)^{1-\beta}}{z^{1-\beta}} + \frac{\dot{p}_N}{p_N} \tag{8}$$

where the total private returns for both types of capital must be the same. Thus, equation (29) states that the private marginal product of $K_M$ is equal to the private marginal product of $K_N$ plus capital gains on $K_N$. We assume that $\alpha > \beta$, so the manufacturing sector is more capital intensive than the non-manufacturing sector.

Using equations (15) and (25), we can define the growth rate of aggregate consumption as:

$$\frac{\dot{C}}{C} = \sigma \left[ \frac{A_M \alpha n^{1-\alpha}}{(1-\mu)} - (1-\gamma) \frac{\dot{p}_N}{p_N} - \rho \right] \tag{30}$$

where $\dot{C}/C = g_C$ is the growth rate of $C$. Alternatively, with equations (15) and (27), we can obtain the growth rate of aggregate consumption as:

$$\frac{\dot{C}}{C} = \sigma \left[ \frac{A_N \beta (1-n)^{1-\beta}}{z^{1-\beta}} + \gamma \frac{\dot{p}_N}{p_N} - \rho \right] \tag{9}$$

Finally, we can rewrite the equilibrium conditions (19) and (20) in terms of the stationary variables. Considering the production function of the manufacturing sector, equation (22), the definition of $v = C / K_N$, the level of $C_M$, equation (16), the identity $I_M = \dot{K}_M$ and that $p_C = p_N^{1-\gamma}$, we can rewrite the equilibrium condition for the market of the manufacturing good, equation (20), as:

$$\frac{\dot{K}_M}{K_M} = A_M n^{1-\alpha} - \gamma p_N^{1-\gamma} v z \tag{32}$$

where $\dot{K}_M/K_M = g_{K_M}$ is the growth rate of $K_M$. Also, with the production function of the non-manufacturing sector, equation (23), the level of $C_N$, equation (17), the identity $I_N = \dot{K}_N$ and that $p_C = p_N^{1-\gamma}$, we can rewrite the equilibrium condition for the market of the non-manufacturing good, equation (19), as:

$$\frac{\dot{K}_N}{K_N} = \frac{A_N(1-n)^{1-\beta}}{z^{1-\beta}} - \frac{(1-\gamma)v}{p_N^{\gamma}} \tag{10}$$

where $\dot{K}_N/K_N = g_{K_N}$ is the growth rate of $K_N$.

## 2.7 THE DYNAMIC SYSTEM IN THE MARKET ECONOMY

We have a dynamic system with three stationary variables, $z, n$ and $v$. We now proceed to form a dynamic system in terns of these variables, that is:

$$\begin{aligned} \dot{z} &= f_1(z,n,v) \\ \dot{n} &= f_2(z,n,v) \\ \dot{v} &= f_3(z,n,v) \end{aligned} \tag{34}$$

where $f_{1,}$ $f_2$ and $f_3$ are non-linear functions. Using the definition of $z$, the growth rate of $z$ is:

$$\frac{\dot{z}}{z} = \frac{\dot{K}_N}{K_N} - \frac{\dot{K}_M}{K_M} \tag{35}$$

Next, we can obtain the growth rates of $K_M$ and $K_N$ in terms of $z$, $n$, $v$ and parameters. From the efficient allocation condition for labor market, equation (28), we can obtain the level of $p_N$ in terms of stationary variables:

$$p_N = \frac{A_M(1-\alpha)(1-n)^{\beta}}{A_N z^{\beta}(1-\beta)n^{\alpha}} \tag{36}$$

Using the previous equation (36), we can rewrite equations (32) and (33) as:

$$\frac{\dot{K}_M}{K_M} = A_M n^{1-\alpha} - \gamma \left[\frac{A_M(1-\alpha)(1-n)^{\beta}}{A_N z^{\beta}(1-\beta)n^{\alpha}}\right]^{1-\gamma} v\, z \tag{37}$$

$$\frac{\dot{K}_N}{K_N} = \frac{A_N(1-n)^{1-\beta}}{z^{1-\beta}} - (1-\gamma)\left[\frac{A_N z^{\beta}(1-\beta)n^{\alpha}}{A_M(1-\alpha)(1-n)^{\beta}}\right]^{\gamma} v \tag{38}$$

Thus, the growth rate of $z$, equation (35), is defined by equations (37) and (38).

Next, we can obtain the growth rate of the stationary variable $n$. Taking logarithms and time derivatives of both sides of the efficient allocation of labor, equation (28), we obtain:

$$\frac{\dot{n}}{n} = \frac{(1-n)}{[\alpha(1-n)+\beta n]}\left[-\frac{\dot{p}_N}{p_N} - \beta\frac{\dot{z}}{z}\right] \tag{39}$$

Using the dynamic arbitrage condition for the two capital goods, equation (29), we can obtain:

$$\frac{\dot{p}_N}{p_N} = \frac{A_M \alpha n^{1-\alpha}}{(1-\mu)} - \frac{A_N \beta (1-n)^{1-\beta}}{z^{1-\beta}} \tag{40}$$

Thus, with the previous equation (40), the growth rate of $n$ can be rewritten as:

$$\frac{\dot{n}}{n} = \frac{(1-n)}{[\alpha(1-n)+\beta n]}\left[\frac{A_N\beta(1-n)^{1-\beta}}{z^{1-\beta}} - \frac{A_M\alpha n^{1-\alpha}}{(1-\mu)} - \beta\left(\frac{\dot{K}_N}{K_N} - \frac{\dot{K}_T}{K_T}\right)\right] \tag{41}$$

where $g_{K_M}$and $g_{K_N}$are given by (37) and (38). Next, we know that the growth rate of the stationary variable $v$ is given by:

$$\frac{\dot{v}}{v} = \frac{\dot{C}}{C} - \frac{\dot{K}_N}{K_N} \tag{42}$$

Substituting equation (40) in equation (30) or (31), we obtain the growth rate of consumption:

$$\frac{\dot{C}}{C} = \sigma\left[\gamma\frac{A_M\alpha n^{1-\alpha}}{(1-\mu)} + (1-\gamma)\frac{A_N\beta(1-n)^{1-\beta}}{z^{1-\beta}} - \rho\right] \tag{43}$$

then the growth rate of $v$ is given by equations (42), (43) and (38).

Therefore, our dynamic system (34) is formed by equations (35), (37), (38), (41), (42) and (43). We can see that the system only depends on $z$, $n$, $v$ and parameters.

Finally, it can be shown that the growth rate of the total output, $g_Y$, is:

$$\frac{\dot{Y}}{Y} = \frac{Y_M}{Y}\frac{\dot{Y}_M}{Y_M} + \frac{P_N Y_N}{Y}\left[\frac{\dot{Y}_N}{Y_N} + \frac{\dot{p}_N}{p_N}\right] \tag{44}$$

where $Y_M/Y = 1/\{1 + [p_N A_N z^\beta (1-n)^{1-\beta}/A_M n^{1-\alpha}]\}$ is the share of $Y_M$ in the value of total output and $p_N Y_N/Y = 1/\{[A_M n^{1-\alpha}/(p_N A_N z^\beta (1-n)^{1-\beta})] + 1\}$ is the share of $p_N Y_N$ in the value of total output. The growth rate of $Y_M$, $g_{Y_M}$, and $Y_N$, $g_{Y_N}$, are given by:

$$\frac{\dot{Y}_M}{Y_M} = \frac{\dot{K}_M}{K_M} + (1-\alpha)\frac{\dot{n}}{n} \tag{45}$$

$$\frac{\dot{Y}_N}{Y_N} = \beta\frac{\dot{z}}{z} + \frac{\dot{K}_M}{K_M} - (1-\beta)\frac{\dot{n}}{n}\frac{n}{(1-n)} \tag{46}$$

## 2.8 THE STEADY STATE SOLUTION AND TRANSITIONAL DYNAMICS IN THE MARKET ECONOMY

We can obtain in the steady state a system of three non-linear equations in three variables, $z$, $n$ and $v$. In the steady state, the growth rate of $z$ is zero, so $g^*_{K_M} = g^*_{K_N}$. The steady state levels are denoted with $*$. Using equations (37) and (38), we have:

$$A_M n^{*(1-\alpha)} - \gamma \left[\frac{A_M(1-\alpha)(1-n^*)^\beta}{A_N z^{*\beta}(1-\beta)n^{*\alpha}}\right]^{1-\gamma} v^* z^*$$
$$= \frac{A_N(1-n^*)^{1-\beta}}{z^{*(1-\beta)}} - (1-\gamma)\left[\frac{A_N z^{*\beta}(1-\beta)n^{*\alpha}}{A_M(1-\alpha)(1-n^*)^\beta}\right]^\gamma v^* \tag{47}$$

In the steady state, the growth rate of $v$ is zero, so $g_C^* = g_{KN}^*$. With $\dot{p}_N / p_N = 0$, equations (30) and (38), we obtain:

$$\sigma\left[\frac{A_M \alpha n^{*(1-\alpha)}}{(1-\mu)} - \rho\right] = \frac{A_N(1-n^*)^{1-\beta}}{z^{*(1-\beta)}} - (1-\gamma)\left[\frac{A_N z^{*\beta}(1-\beta)n^{*\alpha}}{A_M(1-\alpha)(1-n^*)^\beta}\right]^\gamma v^* \tag{48}$$

alternatively, with $\dot{p}_N / p_N = 0$, equations (31) and (38), we have:

$$\sigma\left[\frac{A_N \beta(1-n^*)^{1-\beta}}{z^{*(1-\beta)}} - \rho\right] = \frac{A_N(1-n^*)^{1-\beta}}{z^{*(1-\beta)}} - (1-\gamma)\left[\frac{A_N z^{*\beta}(1-\beta)n^{*\alpha}}{A_M(1-\alpha)(1-n^*)^\beta}\right]^\gamma v^* \tag{49}$$

Given that $\dot{p}_N / p_N = 0$, the dynamic arbitrage condition for the two capital goods, equation (29), is:

$$\frac{A_M \alpha n^{*(1-\alpha)}}{(1-\mu)} = \frac{A_N \beta(1-n^*)^{1-\beta}}{z^{*(1-\beta)}} \tag{50}$$

Therefore, we have obtained a system of three non-linear equations, (47), (48) or (49), and (50), in three variables, $z$, $n$ and $v$. Finally, given that $\dot{p}_N / p_N = 0$ and $\dot{n} = 0$, we can show that the growth rate of $Y$, $g_Y$, is:

$$g_Y^* = \frac{Y_M}{Y} g_{Y_M}^* + \frac{P_N Y_N}{Y} g_{Y_N}^* = g_{K_M}^* \tag{51}$$

We obtain in the steady state:

$$g^* = g_Y^* = g_{Y_M}^* = g_{Y_N}^* = g_{K_M}^* = g_{K_N}^* = g_C^* \tag{52}$$

so $Y$, $Y_M$, and $Y_N$ grows at the same rate as $K_M$, $K_N$ and $C$. Thus, in the steady state, the long run growth rate is defined as $g^*$.

We solve numerically the system of equations, (47), (48) or (49) and (50), with fsolve/MATLAB. Roe, Smith and Saracoğlu (2010) show numerical algorithms for the solution of some multi-sector growth models. We use the following parameter values: Valentinyi and Herrendorf (2008) show (US economy) that the tradable sector (agriculture, manufactured consumption, and equipment investment) is more capital intensive than the non-tradable sector (services and construction investment), thus $\alpha$ = 0.37 and $\beta$ = 0.32. We use $\rho$ = 0.02 as in Barro and Sala-i-Matin (2004). We set $\gamma$ = 0.4 (see Rabanal and Tuesta, 2013). We give $\sigma$ = 0.2 (see Yogo, 2004). As the magnitude of $A_M$ and $A_N$ depend on the unique characteristics of an economy, they are set only for explanatory purposes as $A_M$ = 0.4 and $A_N$ = 0.4. For the moment, we impose $\mu$ = 0. We obtain that $z^*$ = 1.20, $n^*$ = 0.383, $v^*$ = 0.411, $p_N^*$ = 1.06 and $g^*$ = 0.012. Thus, the steady state growth rate is 1.2% per annum.

In order to find a numerical solution of the system of differential equations, system (34), we use the time-elimination method (see Mulligan and Sala-i-Martin, 1991 and 1993). We can obtain a system of equations describing policy functions for $v$ and $n$. Policy function consists of a functional relationship between $n$ and $z$, and between $v$ and $z$, where the time component has been eliminated. The two policy functions are:

$$\begin{aligned} n'(z) &= \frac{dn}{dz} = \frac{\dot{n}}{\dot{z}} = n[z, n(z), v(z)] \\ v'(z) &= \frac{dv}{dz} = \frac{\dot{v}}{\dot{z}} = v[z, n(z), v(z)] \end{aligned} \tag{53}$$

where $\dot{C}/C$, $\dot{K}_M/K_M$ and $\dot{K}_N/K_N$ are given by equations (43), (37) and (38), respectively. The system (53) give the slope of the policy functions for all values of $z$, except in the steady state, given that $v'(z) = 0/0$ and $n'(z) = 0/0$. We can calculate the slope of the policy function in the steady state through the eigenvectors of the system (34). Thus, we can linearize the system (34) around the steady state and with the eigenvalues distinguish stable and unstable arms. Also, with the eigenvectos of (34), we can find the slopes of the authentic stable arms of the policy functions in the steady state. After this, we can solve the system (53) numerically subject to the steady state slopes as an initial value problem, where the initial levels are the steady state levels of the policy functions (the steady state levels of $z$, $n$ and $v$).

We calculate the eigenvalues of the linearization of the system (34) around the steady state and thus obtaining one negative root and two positive roots, this is, the model turns out to be locally saddle path stable. Thus, the variable $z$ is pre-determinate and $n$ and $v$ are jump variables. We apply the time-elimination method and we obtain the two policy functions (we use ODE/MATLAB). In figure 1 and figure 2, we present the policy functions $n = n(z)$ and $v = v(z)$ respectively, without investment subsidy, with levels of $z$ between almost zero and a positive value. We can see that the policy function $n = n(z)$ has a positive slope and the policy function $v = v(z)$ has a negative slope.

Figure 1. The policy function $n(z)$

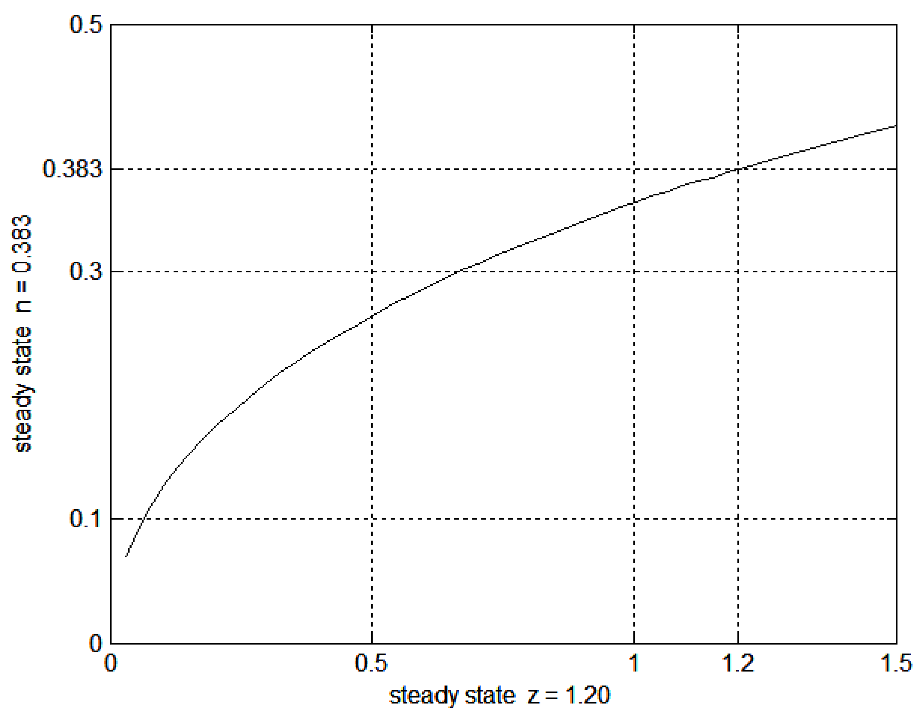
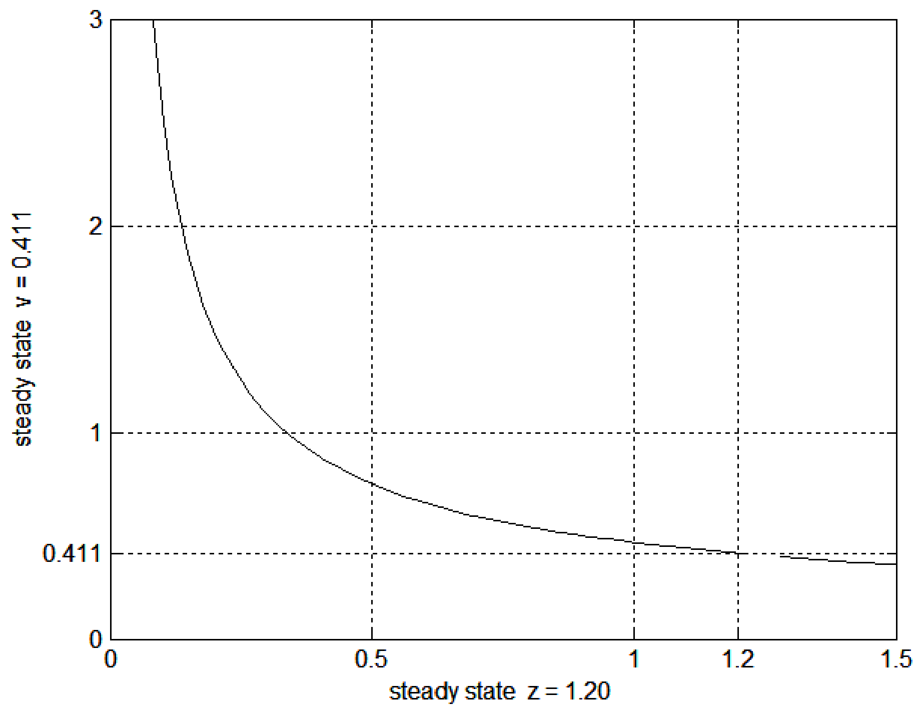


Figure 2. The policy function $v(z)$




## 3. THE COMMAND ECONOMY

Since there are two externalities, the market economy is inefficient. To identify the optimal solution, we need to find the planner´s solution, that is, we need to internalize the externalities. Given that in the command economy there are no markets and prices, the social coordinator maximizes the present value of a constant intertemporal elasticity of substitution utility function:

$$max \;\; U(0) = \int_0^\infty \frac{\left(DC_M^\gamma C_N^{1-\gamma}\right)^{1-1/\sigma}}{1-1/\sigma} e^{-pt} dt$$

subject to $Y_M = C_M + \dot{K}_M$ and $Y_N = C_N + \dot{K}_N$ where $Y_M = A_M K_M n^{1-\alpha}$ and $Y_N = A_N K_N^{\beta} K_M^{1-\beta}(1-n)^{1-\beta}$, which explicitly take into account the externalities and the labor market equilibrium condition.

The Hamiltonian is:

$$H = \left\{ \frac{(DC_M^{\gamma} C_N^{1-\gamma})^{1-1/\sigma}}{1-1/\sigma} + \lambda_M [A_M K_M n^{1-\alpha} - C_M] + \lambda_N \left[ A_N K_N^{\beta} K_M^{1-\beta} (1-n)^{1-\beta} - C_N \right] \right\} e^{-\rho t}$$

$\lambda_M$ and $\lambda_N$ are the shadow prices as of time $t$, of an additional unit of $K_M$ and $K_N$ at time $t$, respectively. The first order conditions with respect to $C_M$, $C_N$ and $n$ are:

$$\left(DC_M^{\gamma} C_N^{1-\gamma}\right)^{-1/\sigma} D\gamma C_M^{\gamma-1} C_N^{1-\gamma} = \lambda_M \tag{54}$$

$$\left(DC_M^{\gamma} C_N^{1-\gamma}\right)^{-\frac{1}{\sigma}} DC_M^{\gamma-1}(1-\gamma) C_N^{-\gamma} = \lambda_N \tag{55}$$

$$A_M K_M (1-\alpha) n^{-\alpha} = \frac{\lambda_N}{\lambda_M} \left[ A_N K_N^{\beta} K_M^{1-\beta} (1-\beta)(1-n)^{-\beta} \right] \tag{56}$$

The first order conditions with respect to $K_M$ and $K_N$ are:

$$A_M n^{1-\alpha} + \frac{\lambda_N}{\lambda_M} \left[ A_N K_N^{\beta} (1-\beta) K_M^{-\beta} (1-n)^{1-\beta} \right] + \frac{\dot{\lambda}_M}{\lambda_M} = \rho \tag{57}$$

$$A_N \beta K_N^{\beta-1} K_M^{1-\beta} (1-n)^{1-\beta} + \frac{\dot{\lambda}_N}{\lambda_N} = \rho \tag{58}$$

with $\lim_{t\to\infty} \lambda_M e^{-\rho t} K_M = 0$ and $\lim_{t\to\infty} \lambda_N e^{-\rho t} K_N = 0$.

Let $p_N = \lambda_N / \lambda_M$ , then we can define aggregate conditions (see Barro and Sala-i-Martin, 2004). Using $z = K_N / K_M$, equation (56) is the static efficient allocation condition for labor: $A_M (1-\alpha) n^{-\alpha} = p_N A_N z^{\beta} (1-\beta)(1-n)^{-\beta}$. We see that the static efficient allocation condition for labor in the command economy is equal to equation (28) in the market economy. Next, substituting $\lambda_N = p_N\lambda_M$ in equation (55), and equating the result in equation (54), we obtain:

$$\frac{\gamma}{(1-\gamma)}\frac{C_N}{C_M} = \frac{1}{p_N} \tag{59}$$

Equation (59) states that the marginal rate of substitution between $C_M$ and $C_N$ is equal to the relative price. With equation (59) and $p_C C = C_M + p_N C_N$, where $p_C = p_N^{1-\gamma} = (\lambda_N/\lambda_M)^{1-\gamma}$ and $\dot{p}_C/p_C = (1-\gamma)\dot{p}_N/p_N$, we obtain the levels of $C_M$ and $C_N$: $C_M = \gamma p_C C$ and $C_N = (1-\gamma)\, p_C C/p_N$. Using equation (54), $C = DC_M^{\gamma}C_N^{1-\gamma}$ and $C_M = \gamma p_C C$, we obtain:

$$C^{-1/\sigma} = \lambda_M p_C \tag{60}$$

With equation (55), $C = DC_M^{\gamma}C_N^{1-\gamma}$ and $C_N = (1-\gamma)\, p_C C/p_N$, we find:

$$C^{-1/\sigma} p_N^{\gamma} = \lambda_N \tag{61}$$

Taking logarithms and time derivatives of equation (60), we obtain $\dot{\lambda}_M/\lambda_M = -(1/\sigma)\,\dot{C}/C - (1-\gamma)\,\dot{p}/p$. Using $z = K_N/K_M$ and equating $\dot{\lambda}_M/\lambda_M$ in equation (57), we have:

$$\frac{\dot{C}}{C} = \sigma\left[A_M n^{1-\alpha} + p_N A_N z^{\beta}(1-\beta)(1-n)^{1-\beta} - (1-\gamma)\frac{\dot{p}_N}{p_N} - \rho\right] \tag{62}$$

alternatively, taking logarithms and time derivatives of equation (61), we have $\dot{\lambda}_N/\lambda_N = -(1/\sigma)\,\dot{C}/C + \gamma\,\dot{p}/p$. Using $z = K_N/K_M$ and equating $\dot{\lambda}_N/\lambda_N$ in equation (58), we obtain:

$$\frac{\dot{C}}{C} = \sigma\left[A_N \beta z^{\beta-1}(1-n)^{1-\beta} + \gamma\frac{\dot{p}_N}{p_N} - \rho\right] \tag{63}$$

Equating equations (62) and (63), we obtain the optimal dynamic arbitrage condition for the two capital goods:

$$A_M n^{1-\alpha} + p_N A_N z^{\beta}(1-\beta)(1-n)^{1-\beta} = A_N \beta z^{\beta-1}(1-n)^{1-\beta} + \frac{\dot{p}_N}{p_N} \qquad (64)$$

indicating that the total social return of $K_M$ is equal to the total social return of $K_N$. When the externalities are internalized, the total social return of $K_M$ is formed by the social marginal product of $K_M$ in the manufacturing sector plus the social marginal product of $K_M$ in the non-manufacturing sector, all expressed relative to the price of the manufacturing good. The total social return of $K_N$ is equal to the social marginal product of $K_N$ plus capital gains. When we compare equation (64) with equation (29) with cero subsidy, and $0 < \alpha < 1$, we conclude that the private return of $K_M$ ($A_M \alpha n^{1-\alpha}$) is lower than the total social return of $K_M$ ($A_M n^{1-\alpha} + p_N A_N z^{\beta}(1-\beta)(1-n)^{1-\beta}$). Thus the market economy is under-accumulating implying than the market economy has a lower growth rate than the optimal growth rate.

### 3.1 DYNAMIC SYSTEM IN THE COMMAND ECONOMY

Next, we need to form a dynamic system as already presented in (34). Accordingly, with the static allocation condition for labor in terms of stationary variables, we obtain the level of $p_N$, as equation (36). The growth rate of $K_M$ is analogous to equation (37) and the growth rate of $K_N$ is similar to equation (38). With the dynamic arbitrage condition for the two capital goods, equation (64), and the level of $p_N$, equation (36), we obtain the growth rate of $p_N$:

$$\frac{\dot{p}_N}{p_N} = A_M n^{1-\alpha} + \frac{A_M(1-\alpha)(1-n)}{n^{\alpha}} - A_N \beta z^{\beta-1}(1-n)^{1-\beta} \qquad (65)$$

Using equations (62) and (65), or alternatively with equations (63) and (65), we can obtain the growth rate of consumption:

$$\frac{\dot{C}}{C}=\sigma\left[\gamma A_M n^{1-\alpha}+\gamma\frac{A_M(1-\alpha)(1-n)}{n^{\alpha}}+(1-\gamma)\frac{A_N\beta(1-n)^{1-\beta}}{z^{1-\beta}}-\rho\right] \quad (66)$$

Taking logarithms and time derivatives of the efficient allocation of labor and using equation (65), we obtain:

$$\frac{\dot{n}}{n}=\frac{(1-n)}{[\alpha(1-n)+\beta n]}\left[A_N\beta z^{\beta-1}(1-n)^{1-\beta}-A_M n^{1-\alpha}-\frac{A_M(1-\alpha)(1-n)}{n^{\alpha}}-\beta\left(\frac{\dot{K}_N}{K_N}-\frac{\dot{K}_M}{K_M}\right)\right] \quad (67)$$

Finally, the dynamic system for the planned economy, as (34), is formed by equations $\dot{z}/z=\dot{K}_N/K_N-\dot{K}_M/K_M$, (37), (38), $\dot{v}/v=\dot{C}/C-\dot{K}_N/K_N$, (66) and (67).

## 3.2 THE STEADY STATE SOLUTION AND TRANSITIONAL DYNAMICS IN THE COMMAND ECONOMY

Now, we solve the command economy in the steady state. The growth rate of $K_M$ is given by equation (37) and the growth rate of $K_N$ is given by equation (38). In the steady state, $g_{K_M}^*=g_{K_N}^*$, so we again obtain equation (47). Next, we know that the growth rate of $v$ is $\dot{v}/v=\dot{C}/C-\dot{K}_N/K_N$. Given that $\dot{p}_N/p_N=0$, and using $p_N$, equation (36), the growth rate of $C$, equation (62), in the steady state is:

$$g_C^*=\sigma\left[A_M n^{*(1-\alpha)}+\frac{A_M(1-\alpha)(1-n^*)}{n^{*\alpha}}-\rho\right] \quad (68)$$

alternatively, with equations (63), we obtain:

$$g_C^* = \sigma\left[A_N \beta z^{*(\beta-1)}(1-n^*)^{1-\beta} - \rho\right] \tag{69}$$

In the steady state, $g_C^* = g_{KN}^*$, from equations (68) and (38), we have:

$$\sigma\left[A_M n^{*(1-\alpha)} + \frac{A_M(1-\alpha)(1-n^*)}{n^{*\alpha}} - \rho\right] = \frac{A_N(1-n^*)^{1-\beta}}{z^{*(1-\beta)}} - (1-\gamma)\left[\frac{A_N z^{*\beta}(1-\beta)n^{*\alpha}}{A_M(1-\alpha)(1-n^*)^{\beta}}\right]^{\gamma} v^* \tag{70}$$

alternatively, with equations (69) and (38):

$$\sigma\left[A_N \beta z^{*(\beta-1)}(1-n^*)^{1-\beta} - \rho\right] = \frac{A_N(1-n^*)^{1-\beta}}{z^{*(1-\beta)}} - (1-\gamma)\left[\frac{A_N z^{*\beta}(1-\beta)n^{*\alpha}}{A_M(1-\alpha)(1-n^*)^{\beta}}\right]^{\gamma} v^* \tag{71}$$

Given that $\dot{p}_N / p_N = 0$, and using $p_N$, equation (36), the dynamic arbitrage condition for the two capital goods is:

$$A_M n^{*(1-\alpha)} + \frac{A_M(1-\alpha)(1-n^*)}{n^{*\alpha}} = A_N \beta z^{*(\beta-1)}(1-n^*)^{1-\beta} \tag{72}$$

We obtain a system of three non-linear equations, (47), (70) or (71), and (72), in three variables, $z$, $n$ and $v$ and parameters. Next, using the parameter values of Section 2, we solve the dynamic system for the command economy in the steady state, obtaining $z^*$ = 0.091, $n^*$ = 0.464, $v^*$ = 2.83, $p_N^*$ = 2.16, and $g^*$ = 0.081. We can see that the steady state optimal growth rate is 8.1% per annum. When we compare the optimal steady state growth rate with the steady state growth rate of the market economy, with $\mu$ = 0, we deduce that there is opportunity for improving the steady state growth rate in the market economy. Thus, the government can increase the steady state growth rate. The correct policy to achieve the optimal steady state growth rate is through an investment subsidy in the manufacturing sector.

Using the time-elimination method, we can solve the dynamic model for the planned economy. We first calculate the eigenvalues of the linerization of the dynamic system for the command economy around the steady state and we obtain one negative eigenvalue and two positive eigenvalue, that is, the model of the planned economy turns out to be locally saddle path stable. We next apply the time-elimination method and we show in figure 3 the optimal policy function $n = n(z)$; it has a positive slope. Note that the corresponding policy function in the market economy, without investment subsidy, also has a positive slope. In figure 4, we present the optimal policy function $v = v(z)$; it has a negative slope. Note that the corresponding policy function in the market economy also has a negative slope. The government can intervene in the market economy to reproduce the optimal policy functions through an investment subsidy in the manufacturing sector.

Figure 3. The policy function $n(z)$

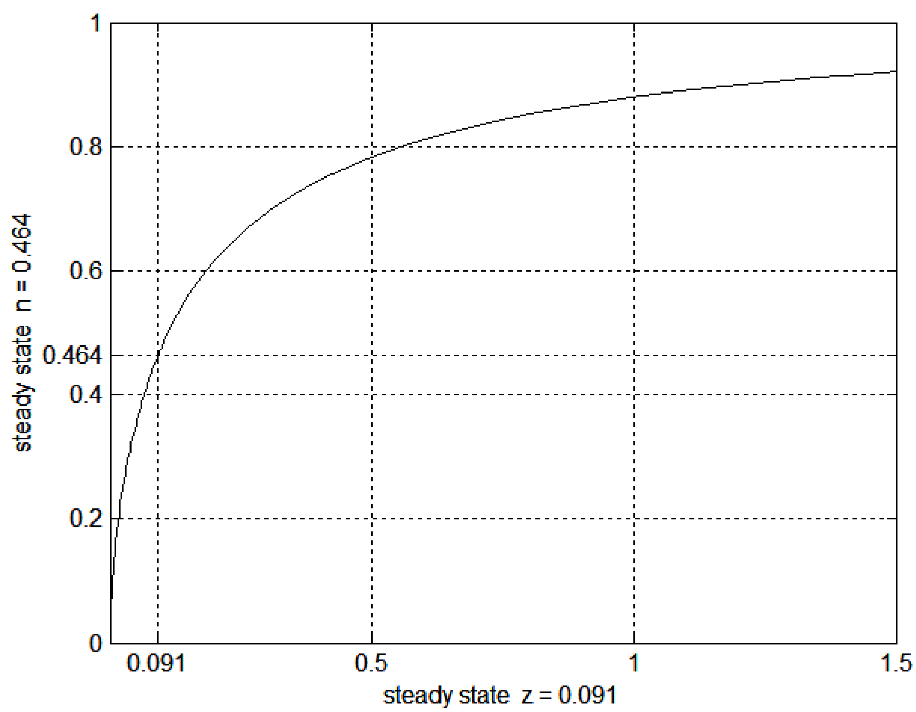
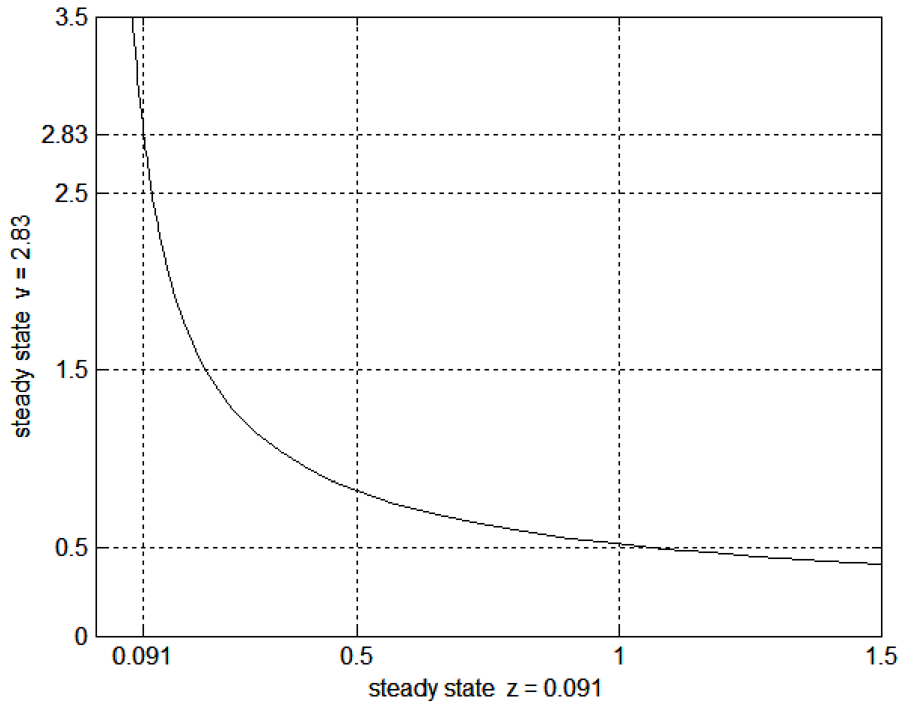


Figure 4. The policy function $v(z)$




## 4. THE OPTIMAL INVESTMENT SUBSIDY IN THE MARKET ECONOMY

The objective of the government in a market economy is to maximize social welfare and to reach the optimal growth rate. The optimal government policy is to establish an investment subsidy in the manufacturing sector, stimulating the source of the learning process.

Using the optimal steady state solution and equation (43), the optimal investment subsidy in the steady state is $\mu = 0.785$. Using this optimal investment subsidy, we solve the system for $z$, $n$ and $v$ in the steady state, equations (47), (70) or (71), and (72). We obtain $z^* = 0.091$, $n^* = 0.464$, $v^* = 2.83$, $p_N^* = 2.16$ and $g^* = 0.081$. Thus, the steady state growth rate is 8.1% per annum. Note that all these levels correspond to the optimal solution.

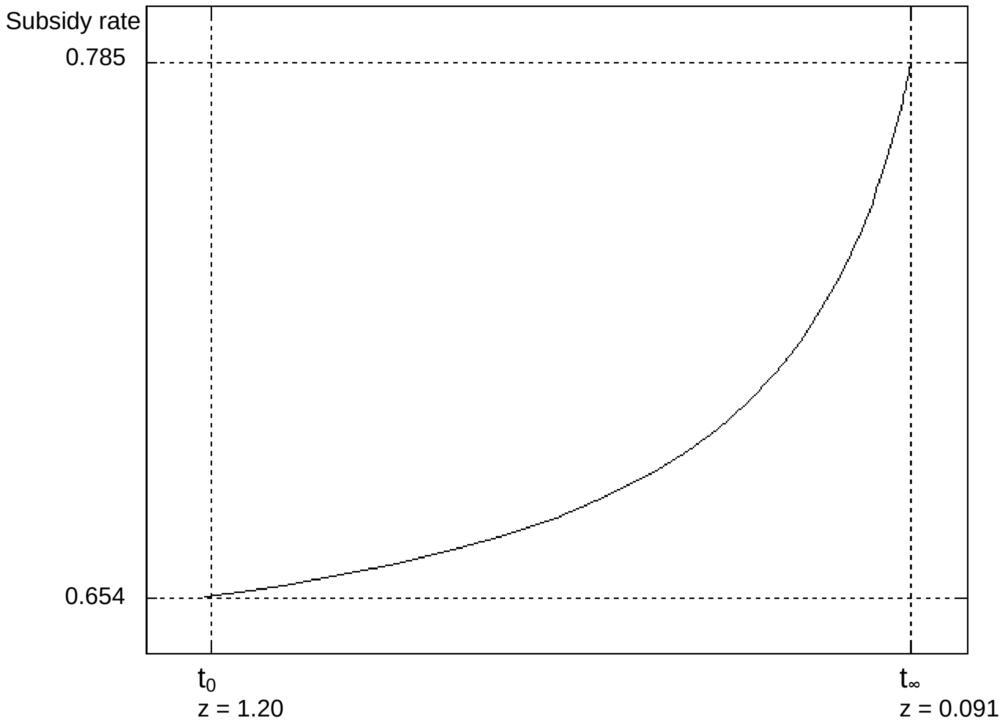
Now, we are ready to analyze how the variables of the economy respond to an increase in the rate of investment subsidy. First, using equations (28) and (50), we can obtain a useful relationship in the steady state:

$$n^* = \frac{1}{(1-\mu)^{\beta/(\alpha-\beta)}} \frac{1}{p_N^{*(1-\beta)/(\alpha-\beta)}} \left[\frac{A_M \alpha}{A_N \beta}\right]^{\beta/(\alpha-\beta)} \left[\frac{A_M(1-\alpha)}{A_N(1-\beta)}\right]^{(1-\beta)/(\alpha-\beta)} \tag{73}$$

Next, we show the response of the variables when the government establishes the optimal rate of investment subsidy. Considering that $p_N^*$ is constant for the moment, and that $\alpha > \beta$, we can see in equation (73) that when $\mu$ increases, the manufacturing sector is stimulated, and the proportion of labor in the manufacturing sector increases initially. Likewise, the incentive to invest (disinvest) in the manufacturing (non-manufacturing) sector increases (decreases). Consequently, the level of $z$ decreases slowly. Also, as the relative price of the non-manufacturing good is flexible, we can see in equation (36) that when $n$ increases, the relative price decreases initially. This confirms the initial increase in $n$ when $\mu$ increases (see equation, 73). However, in the optimal steady state, the level of the relative price of the non-manufacturing good is higher. Moreover, given that total wealth increases, the level of $v$ increases. Therefore, in the optimal steady state, the level of $z^*$ decreases from 1.20 to 0.091, the proportion of labor in the manufacturing sector increases from 0.383 to 0.464, $v^*$ increases from 0.411 to 2.83 and $p_N^*$ increases from 1.06 to 2.16. Therefore, as the manufacturing sector is the leading sector in technological terms, the economy has a higher growth rate. The growth rate increases from 1.2% to 8.1% per annum.

Using the policy function $n(z)$ of the command economy, we obtain the time path of the optimal investment subsidy. In figure 5, we show the optimal path of the subsidy when the economy moves to the optimal steady state. In order to reproduce the optimal path, the government imposes initially, with z = 1.20, a rate of subsidy of 0.654. Thus, as $z$ moves to the optimal steady state ($z$ = 0.091), the subsidy rate increases until $\mu$ = 0.785, the steady state level. Then, using the optimal path of the investment subsidy, we apply the time-elimination method and obtain the policy functions $v = v(z)$ and $n = n(z)$. We can deduce that the policy functions are identical to the policy functions of the planned solution. Thus, the government reproduces the optimal policy functions.

Figure 5. Optimal subsidy




**E) Reflexiones Finales:**

## 5. CONCLUSIONS

We have studied an economy with manufacturing and non-manufacturing goods with two externalities. The relative price of the non-manufacturing good is endogenously determined by supply and demand for the non-manufacturing good. We have also shown that the optimal growth rate is achieved with an investment subsidy in the manufacturing sector.

We have studied how the economy responds, in the steady state, when the government establishes the optimal investment subsidy. When the rate of subsidy is increased, the manufacturing sector is stimulated. Thus, the proportion of labor in the manufacturing sector increases, and the proportion of labor in the non-manufacturing sector decreases. Likewise, investment in the manufacturing sector increases, and investment decreases in the non-manufacturing sector. Thus, the ratio of non-manufacturing to manufacturing capital decreases slowly. In addition, given that the relative price of the non-manufacturing good is flexible, the relative price decreases initially. Also, this relative price adjustment produces a further initial increase in the proportion of labor in the manufacturing sector. Nevertheless, the relative price of the non-manufacturing good is higher in the optimal steady state. Also, given that total wealth increases, the ratio of aggregate consumption to non-manufacturing capital increases.

In summary, in the optimal solution, the proportion of labor in the manufacturing sector, the relative price of the non-manufacturing good, and the ratio of consumption to non-manufacturing capital are higher, and the ratio of non-manufacturing to manufacturing capital is lower. Therefore, as the manufacturing sector is leader in technological terms, the market economy has a higher growth rate. We also have obtained the policy functions for the market economy and the command economy. Thus, with the optimal solution, we obtain the time path of the optimal investment subsidy. The optimal investment subsidy is increasing while the economy moves to the optimal steady state.

Thus, if the economy is technologically commanded by the manufacturing sector and there is strong intra and inter learning by doing among firms and sectors, the government should establish an optimal investment subsidy in the manufacturing sector. Thus, this paper has presented in an overall manner a general conclusion, concerning models with production externalities, two types of capital and endogenous growth: that the optimal policy is to stimulate the sources of the learning process (see Bardhan, 1993). However, if subsidies are permitted or not, or if governments have the ability to manage an economy with externalities or not, there still remains another question that is not solved: how a government can justify subsidizing a particular sector in a democratic society, since these practices certainly have political costs.

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